of Mars. One of these terms involves the eccentricity of the asteroid as a factor, and the other involves the eccentricity of Mars.

4. Long-period inequalities in the major axes arise from the cause mentioned in 1.

5. There are no long-period inequalities in the major axes arising from the action of Jupiter.

6. Long-period inequalities in the eccentricities arise from each of the causes mentioned in 1 and 2. The ranges of variation of those due to the action of Mars are appreciably less than those due to the action of Jupiter.

7. The mean longitude may be expressed in the form

$$L = n_1 t + \epsilon_1 + N_1 t + \text{periodic terms},$$

where n_1 is the initial value of the theoretical mean motion, this being deduced from the initial value of the major axis by Kepler's third law.

This may be written

$$L = Nt + \epsilon_1 + \text{periodic terms}.$$

N may conveniently be called the "real mean motion."

Of the periodic terms of long period in L the amplitudes of those arising from the action of Mars may be appreciably greater than the amplitudes of those arising from the action of Jupiter, and when the approach to exact commensurability of the mean motions is near enough, the phenomena of libration (the cases B in the theory) may manifest themselves, and the amplitude of the corresponding periodic term may be comparatively large. In fact, when k, the modulus of elliptic functions, tends to unity in any of the cases A and B, the amplitude A tends to $\frac{3}{4D_2} \times 90°$, *i.e.* to 90° approx. In the example above we found amplitudes of approximately 40° and 62°.

It is a pleasant duty to acknowledge my indebtedness to Professor Baker for his continued interest in this and the previous investigation.

Invisible Sunspots. By Dr. G. E. Hale.

(*Extract from a letter to Professor Newall,* 1921 *Dec.* 21.)

I have been able to give more time to my own solar work, and the measurement and reduction of the sunspot spectra, so long delayed by various causes, is now well advanced. Another piece of work which I had in mind when with you has also made progress recently.

You may remember that I commented on the strong tendency of spots to appear in the bipolar form, which is so marked that some 60 per cent. of all spots are double, while almost all single spots are followed (sometimes preceded) by streams of calcium flocculi. It struck me that an incipient spot, not dark enough to be visible, might frequently lie in the flocculi, at a point corresponding with the position of the second member of a full-fledged bipolar group.

Lately Ellerman and I have used the Zeeman effect to detect invisible vortices in such regions. Some time ago I found local magnetic fields in the midst of faculæ where no spots were visible, and it struck me that a systematic search for "invisible spots" might be worth while. The best way to find them is to mount a small half-wave plate above the compound quarter-wave plate and nicol ordinarily used over the slit of the spectrograph. The half-wave plate is made by a small electric motor to oscillate back and forth across the slit, thus transmitting alternately the red and violet *n* components of a Zeeman triplet. Promising regions of the sun, such as the calcium flocculi following a single spot, are moved slowly across the slit by the cœlostat motor, while the observer watches the iron line $\lambda 6173$ (the triplet used for our daily observations). The presence of a magnetic field with an intensity of 200 gausses, or even less, is shown by the widening of the line towards red or violet as the half-wave plate alternately covers and uncovers the slit.

The first local field thus found followed a single spot, and had the same polarity and approximate position as the following component of a regular bipolar group. (Near it was another local field of the same polarity.) This remained invisible for three days, and then appeared as an ordinary small spot. On the return of this group in the next solar rotation this spot was no longer visible, but its field and another small field were detected as soon as the group had advanced far enough upon the disc. Two days later both of the small spots became visible. In another case the local field of a very small spot (the following member of a bipolar group) could be detected for two days after the spot had disappeared.

It will be interesting to go on with this work, and especially to look for local fields in isolated faculæ or flocculi where no spots can be seen. I am also anxious to see how long a magnetic field can be detected before or after the appearance or disappearance of a large spot. The effect irresistibly recalls the rise and fall of the free end of my flexible vortex near the water-surface as the speed of the motor varies, but in many cases, at any rate, a bipolar spot cannot be such a simple thing as a semicircular vortex ring. It certainly seems impossible to account in this way for the mixed polarities of the sunspot of last May (the one that produced the magnetic storm and aurora), and the proper motion of a vortex ring is always a serious objection to this hypothesis. Yet if the vortices of a bipolar group penetrate to a sufficient depth they must certainly unite to form a half-ring.

Another thing that interests me is the bearing of invisible spots on the level of origin of the main spot vortex. The calcium (H) flocculi now appear to be the best guide to the existence of invisible spots, but it is probable that faculæ would be equally useful if as easily observed near the centre of the sun. "Invisible spots" may sound like a misnomer, but it seems the simplest name to apply to spots which may be visible during part of their existence.

Terrestrial Magnetic Disturbances and Sun-spots.
By Rev. A. L. Cortie, S.J., F.Inst.P.

One of the chief conclusions stated by Mr. Maunder in his very important paper on "Magnetic Disturbances, 1882 to 1903, as recorded at the Royal Observatory, Greenwich, and their Association with Sun-spots" (*Monthly Notices R.A.S.*, **65**, 23), was to the effect that "the rotation periods given us by the magnetic disturbances not only agree in the mean with the rotation periods given by sun-spots, but the limits within which they vary are the same." The object of the present paper is to extend the investigation to some longer magnetic series of the present solar cycle, and to bring evidence to show that the connection is still closer. In fact, the duration of the magnetic period is in general a function of the latitude of the spot-disturbance. This paper has grown out of a correspondence on the subject in *Nature* (1921 Oct. 6, Oct. 27, Dec. 29; 1922 Jan. 12). It will not be necessary to set out in detail the lengthy series of magnetic disturbances upon which it is founded. These can be reconstructed, from the limiting dates in the third column of the subjoined table, by means of the lists of magnetic disturbances given in the Annual Reports of this observatory.

TABLE.

Terrestrial Magnetic Disturbances and Sun-spots.

				Mean Period.		Solar		
	Year.	Duration.	No. of Days.	Synodic.	Sidereal.	Daily Rotation.	Latitude.	Spot Group.
1	1917	July 12–Nov. 26	137	27·40	25·49	14°·13	±15·8	+16·0
2	1918	July 1–Jan. 8	218	27·25	25·36	14·20	±13·5	+16·1
3	1919	May 24–Oct. 6	135	27·00	25·14	14·32	± 5·7	− 7·8
4	1920	Jan. 1–Nov. 21	325	27·08	25·21	14·28	±10·2	− 5·8
5	1921	March 22–Sept. 29	191	27·29	25·39	14·18	±14·1	+ 9·0
6	1921	May 21–Dec. 24	217	27·13	25·25	14·26	±10·9	?
	Mean sidereal period,				25·31			

For the sun's mean period of rotation Carrington's value is 25·38, and Spoerer's 25·23. The mean of these two numbers is 25·31, coinciding exactly with the mean sidereal period of these magnetic disturbances.

Notes.

1. The associated sun-spot group was the great group of August, recurring in September, Nos. 771, 813 of the Tortosa Boletín Mensuel.

2. The associated sun-spot group, which recurred three times, bears the numbers 1045, 1066, 1088, 1106 of the Tortosa Observatory records.

3. The associated sun-spot groups are 1242, 1262, 1282, 1300, four appearances.

4. The associated sun-spot group appeared six times, Nos. 1370,